

# Jaipur Institute of Technology Group of Institutions



Jaipur - Near Mahindra SEZ Kalwara, Ajmer Road

## MANUAL

### VIII SEM EE

**8EE5**

**COMPUTER BASED POWER SYSTEM  
LAB**

**DEPARTMENT OF ELECTRICAL ENGINEERING**

## **EXPERIMENTS LIST**

### **8EE5 COMPUTER BASED POWER SYSTEM LAB**

1. Fault analysis (for 3 to 6 bus) and verify the results using MATLAB or any available software for the following cases:
  - (i) LG Fault
  - (ii) LLG Fault
  - (iii) LL Fault and
  - (iv) 3-Phase Fault
2. Load flow analysis for a given system (for 3 to 6 bus) using
  - (i) Gauss Seidal
  - (ii) Newton Raphson
  - (iii) Fast Decoupled Method and verify results using MATLAB or any available Software
3. Study of voltage security analysis
4. Study of overload security analysis and obtain results for the given problem using MATLAB or any software.
5. Study of economic load dispatch problem with different methods.
6. Study of transient stability analysis using MATLAB/ETAP Software.

### **LAB ETHICS**

#### ***DO's***

- 1. Enter the lab on time and leave at proper time.**
- 2. Keep the bags outside in the racks.**
- 3. Utilize lab hours in the corresponding experiment.**
- 4. Make the Supply off the Kits/Equipment's after completion of Experiments.**
- 5. Maintain the decorum of the lab.**

#### **Don'ts**

- 1. Don't bring any external material in the lab.**
- 2. Don't make noise in the lab.**

3. Don't bring the mobile in the lab.
4. Don't enter in Faculty room without permission.
5. Don't litter in the lab.
6. Don't carry any lab equipment's outside the lab

We need your full support and cooperation for smooth functioning of the lab.

# **Jaipur Institute of Technology Group of Institutions**

## **DEPARTMENT OF ELECTRICAL ENGINEERING**

### **INSTRUCTIONS**

#### **BEFORE ENTERING IN THE LAB**

1. All the students are supposed to prepare the theory regarding the present Experiment.
2. Students are supposed to bring the practical file and the lab copy.
3. Previous experiment should be written in the practical file.
4. Object, Apparatus Table & Brief Theory of the current practical should be written in the lab copy.
5. Any student not following these instructions will be denied entry in the lab and Sessional Marks will be affected.

#### **WHILE WORKING IN THE LAB**

1. Adhere to experimental schedule as instructed by the faculty. 2. Record the observations in lab copy & checked by the faculty
3. Each student should work on his assigned table of the lab.
4. Take responsibility of valuable accessories.
5. Concentrate on the assigned practical and be careful.

6. **If anyone is caught red-handed carrying any equipment of the lab, then he will have to face serious consequences.**

## **EXPERIMENT # 1**

### **OBJECT:**

Fault Analysis for a 3 bus system with verification of results using MATLAB for the following cases

–

- i. LG Fault
- ii. LL Fault
- iii. LLG Fault iv. 3-phase Fault

### **APPARATUS REQUIRED:**

### **THEORY:**

Short circuits occur in power system due to various reasons like, equipment failure, lightning strikes, falling of branches or trees on the transmission lines, switching surges, insulation failures and other electrical or mechanical causes. All these are collectively called faults in power systems.

A fault usually results in high current flowing through the lines and if adequate protection is not taken, may result in damages in the power apparatus.

### **SYMMETRICAL FAULT:**

In power engineering, specifically three-phase power a symmetric, symmetrical or balanced fault is an electrical fault which affects each of the three-phases equally. In transmission line faults, roughly 5% are symmetric. This is in contrast to an asymmetric fault, where the three phases are not affected equally. In practice, most faults in power systems are unbalanced. With this in mind, symmetric faults can be viewed as somewhat of an abstraction; however, as asymmetric faults are difficult to analyze, analysis of asymmetric faults is built up from a thorough understanding of symmetric faults.

### **ASYMMETRICAL FAULT:**

In power engineering, specifically three phase power, an **asymmetric** or **unbalanced fault** is a fault which does not affect each of the three phases equally. This is in contrast to asymmetric fault, where each of the phases is affected equally. In practice, most faults in power systems are unbalanced;

however, as asymmetric faults are difficult to analyze, analysis of asymmetric faults is built up from a thorough understanding of symmetric faults.

Common types of asymmetric faults, and their causes:

- ✦ **Line-to-line** - a short circuit between lines, caused by ionization of air, or when lines come into physical contact, for example due to a broken insulator.
- ✦ **Line-to-ground** - a short circuit between one line and ground, very often caused by physical contact, for example due to lightning or other storm damage
- ✦ **Double line-to-ground** - two lines come into contact with the ground (and each other), also commonly due to storm damage.

### A. SINGLE-LINE-TO-GROUND FAULT

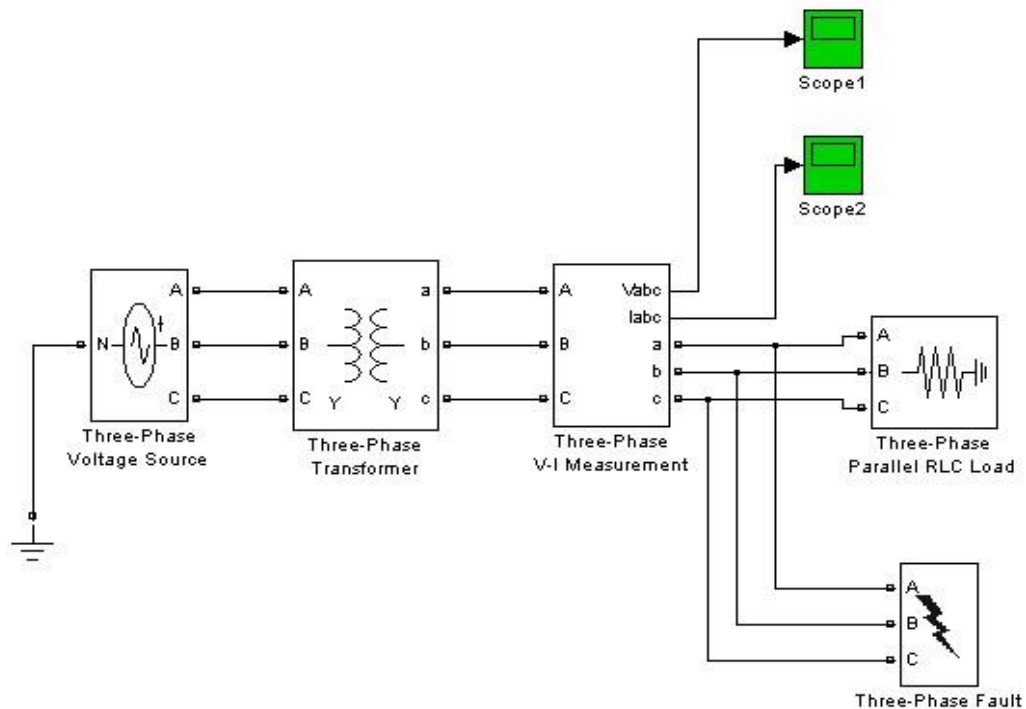


Fig: 1.1-Three phase fault Analysis

|                                  |                   |
|----------------------------------|-------------------|
| Faulted Phase                    | : Phase to Ground |
| Transition state                 | : 1 0 1           |
| Transition Time                  | : 0 0.05 0.1 0.2  |
| Nominal $\phi$ to $\phi$ voltage | : 220 V           |
| Active Power                     | : 100 W           |
| Nominal Frequency                | : 50 Hz           |

Let a LG fault has occurred at node  $k$  of a network. The faulted segment is then as shown in Fig. 1.1 where it is assumed that phase-a has touched the ground through an impedance  $Z_f$ . Since the

system is unloaded before the occurrence of the fault we have

$$I_{f0} = I_{fc} = 0 \dots\dots\dots 2.1$$

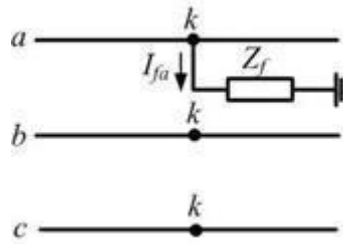


Fig. 1.2 Representation of L – G fault.

Also the phase-a voltage at the fault point is given by

$$V_{ka} = Z_f I_{fa} \dots\dots\dots 2.2$$

From (2.1) we can write

$$I_{fa012} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix} \dots\dots\dots 2.3$$

Solving (2.3) we get

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{I_{fa}}{3} \dots\dots\dots 2.4$$

This implies that the three sequence currents are in series for the LG fault. Let us denote the zero, positive and negative sequence Thevenin impedance at the faulted point as  $Z_{kk0}$ ,  $Z_{kk1}$  and  $Z_{kk2}$  respectively. Also since the Thevenin voltage at the faulted phase is  $V_f$  we get three sequence circuits. We can then write

$$\begin{aligned}
 V_{ka0} &= -Z_{kk0} I_{fa0} \\
 V_{ka1} &= V_f - Z_{kk1} I_{fa1} \\
 V_{ka2} &= -Z_{kk2} I_{fa2}
 \end{aligned}
 \dots\dots\dots 2.5$$

Then from (2.4) and (2.5) we can write

Again since

$$\begin{aligned}
 V_{ka} &= V_{ka0} + V_{ka1} + V_{ka2} \\
 &= V_f - (Z_{kk0} + Z_{kk1} + Z_{kk2}) I_{fa0} \\
 \dots\dots\dots 2.6 \quad V_{ka} &= Z_f I_{fa} = Z_f (I_{fa0} + I_{fa1} + I_{fa2}) = 3Z_f I_{fa0}
 \end{aligned}$$

We get from (2.6)

$$I_{fa0} = \frac{V_f}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f}
 \dots\dots\dots 2.7$$

The Thevenin equivalent of the sequence network is shown in Fig. 1.3

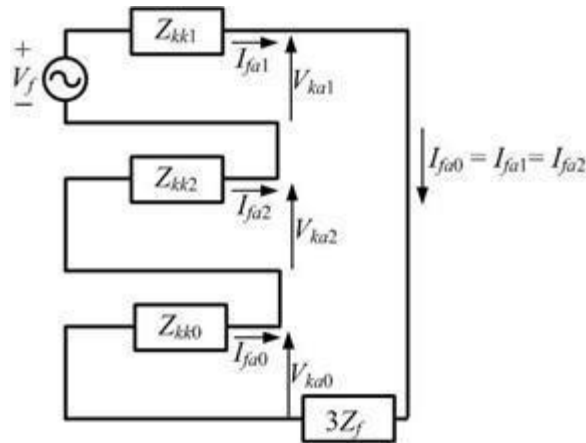


Fig. 1.3 Thevenin equivalent of a L – G fault.

**Result:** We have successfully studied the three phase fault on Line – Ground fault and obtain the voltage and current waveforms shown below;

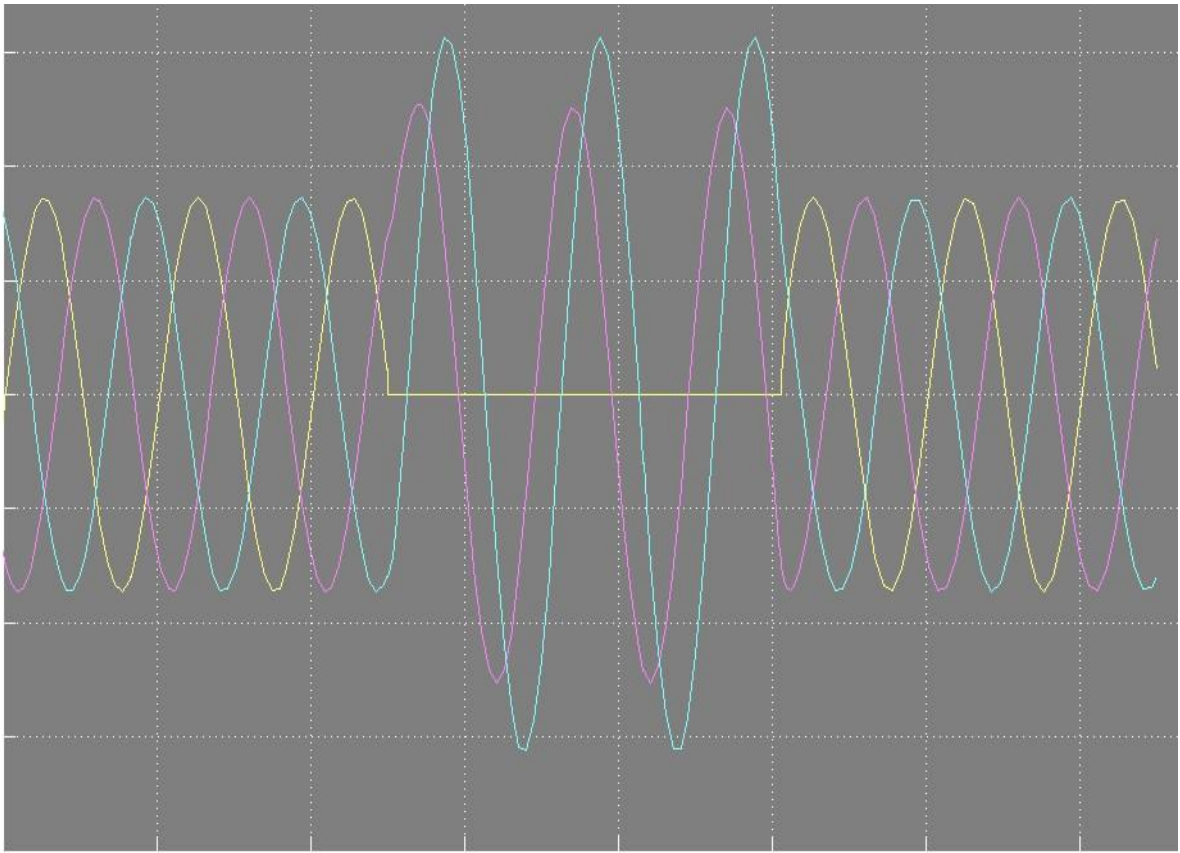


Fig: 1.4 – Voltage Waveform of Line – Ground Fault



Fig: 1.5 – Current Waveform of Line – Ground Fault



## B. LINE-TO-LINE FAULT

This is a type of the asymmetrical faults in this fault two different lines of three phase line touch each other. In above case the line get touch with the line. It occurs in power system due to various reasons like, equipment failure, lightning strikes, falling of branches.

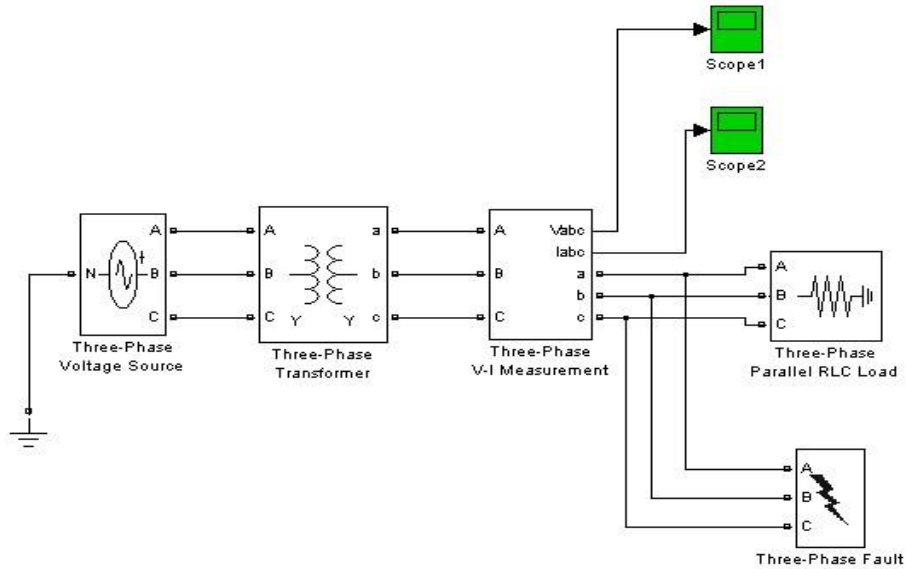


Fig: 1.6 -Three phase fault Analysis

|                                  |                           |
|----------------------------------|---------------------------|
| Faulted Phase                    | : Phase B to C            |
| Transition state                 | : 1 0 1                   |
| Transition Time                  | : 0    0.05    0.1    0.2 |
| Nominal $\phi$ to $\phi$ voltage | : 220 V                   |
| Active Power                     | : 100 W                   |
| Nominal Frequency                | : 50 Hz                   |

The faulted segment for an L-L fault is shown in Fig. 1.6 where it is assumed that the fault has occurred at node  $k$  of the network. In this the phases b and c got shorted through the impedance  $Z_f$ . Since the system is unloaded before the occurrence of the fault we have

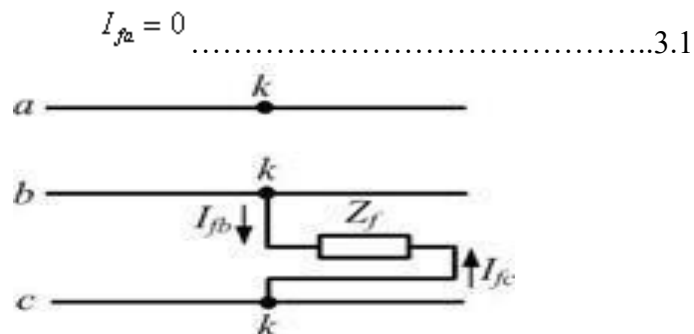


Fig. 1.7 Representation of L-L fault.

Also since phase's b and c are shorted we have

$$I_{fb} = -I_{fc} \quad 3.2$$

Therefore from (3.1) and (3.2) we have

$$3.3 \quad I_{fa012} = C \begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 \\ (a - a^2)I_{fb} \\ (a^2 - a)I_{fb} \end{bmatrix}$$

We can then summarize from (3.3)

$$\begin{aligned} I_{fa0} &= 0 \\ I_{fa1} &= -I_{fa2} \end{aligned} \quad 3.4$$

Therefore no zero

sequence current is injected into the network at bus k and hence the zero sequence remains a dead network for an L-L fault. The positive and negative sequence currents are negative of each other.

Now from Fig. 1.7 we get the following expression for the voltage at the faulted point

$$\text{Again} \quad V_{kb} - V_{kc} = Z_f I_{fb} \quad 3.5$$

$$\begin{aligned} V_{kb} - V_{kc} &= V_{kb0} + V_{kb1} + V_{kb2} - V_{kc0} - V_{kc1} - V_{kc2} \\ &= (V_{kb1} - V_{kc1}) + (V_{kb2} - V_{kc2}) \\ &= (a^2 - a)V_{ka1} + (a - a^2)V_{ka2} \\ &= (a^2 - a)(V_{ka1} - V_{ka2}) \end{aligned} \quad 3.6$$

Moreover

Since  $I_{fa0} = I_{fb0} = 0$  And  $I_{fa1} = -I_{fb2}$ ,

We can write

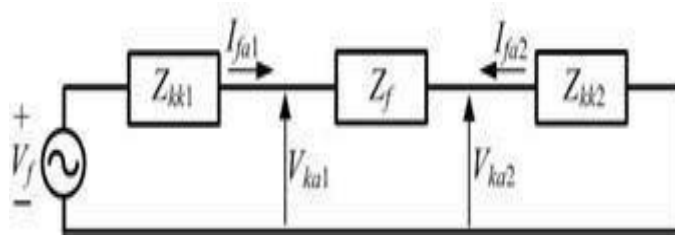
$$3.7 \quad I_{fb} = I_{fb1} + I_{fb2} = a^2 I_{fa1} + a I_{fb2} = (a^2 - a)I_{fa1}$$

Therefore combining (3.5) - (3.7) we get

$$3.8 \quad V_{ka1} - V_{ka2} = Z_f I_{fa1}$$

Equations (3.5) and (3.8) indicate that the positive and negative sequence networks are in parallel. The sequence network is then as shown in Fig. 3.2(A). From this network we get

$$I_{fa1} = -I_{fa2} = \frac{V_f}{Z_{kk1} + Z_{kk2} + Z_f}$$



3.9

Fig. 1.8 Thevenin equivalent of an L – L fault.

**Result:** We have successfully studied the three phase fault on Line – Line fault and obtain the voltage and current waveforms shown below;

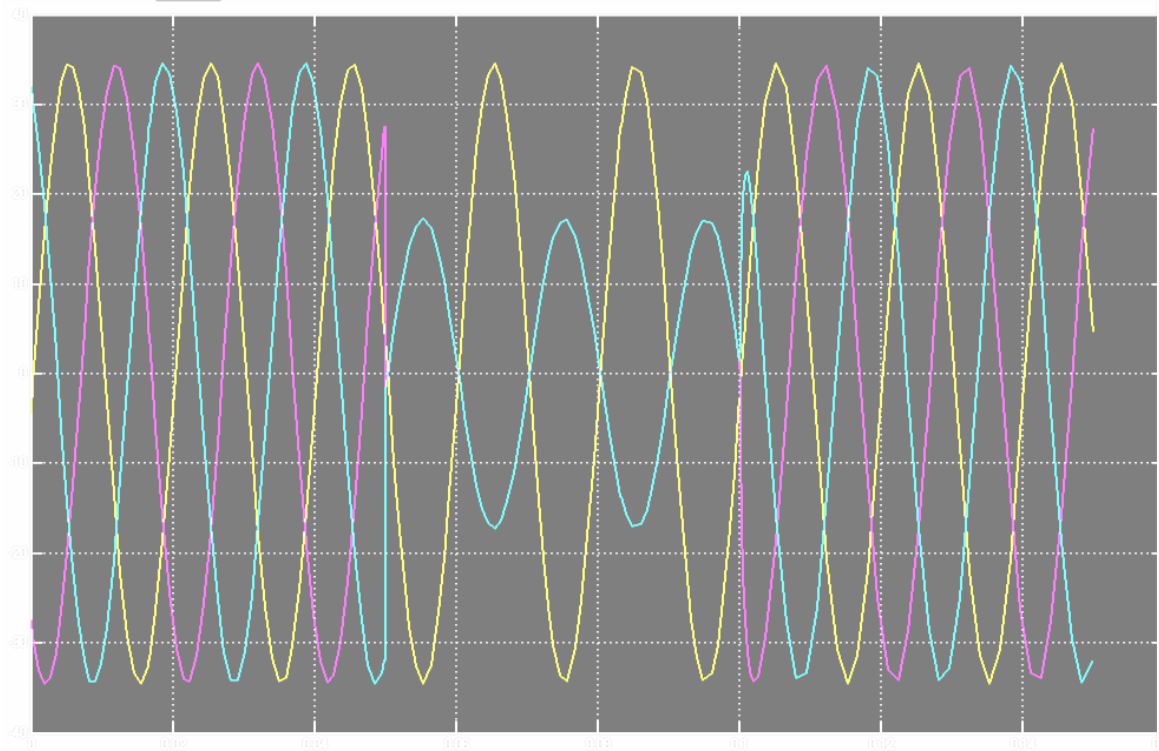


Fig: 1.9–Voltage Waveform of Line – Line Fault

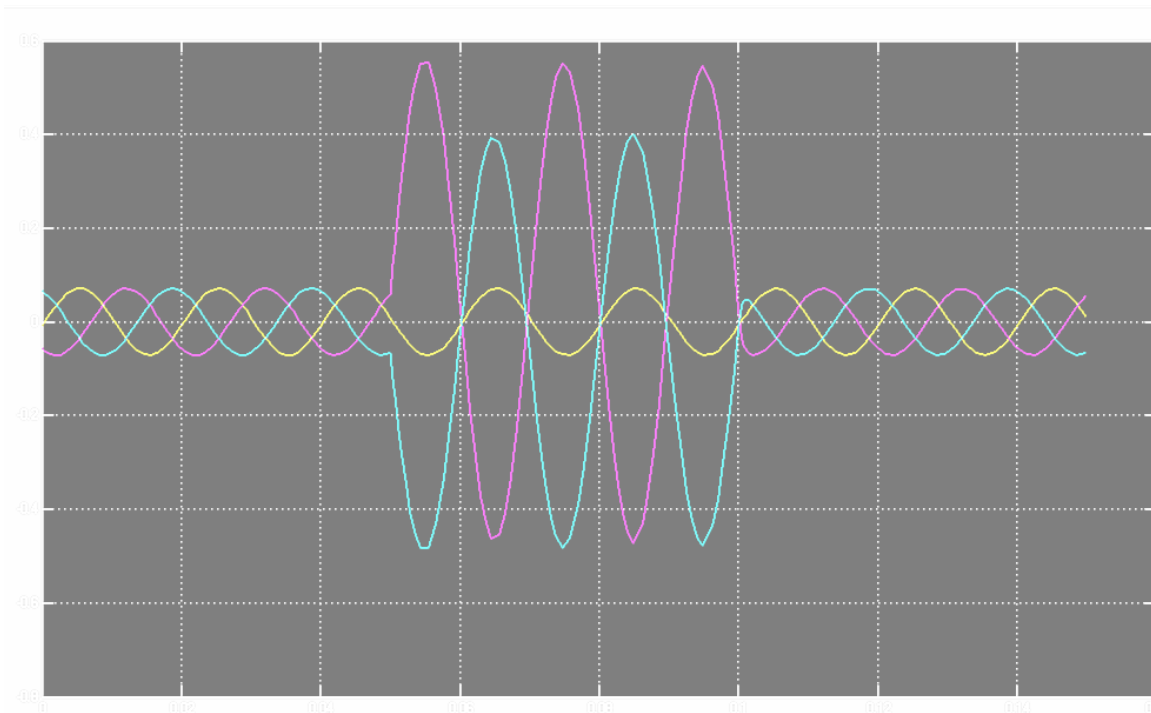


Fig: 1.10 – Current Waveform of Line – Line Fault

### C. DOUBLE - LINE -TO GROUND FAULT

This type of faults occurs when any two lines out of the three phases fell on ground. It is also a type of the asymmetric faults. This type of faults occurs in power system due to various reasons like, equipment failure, lightning strikes, falling of branches etc.

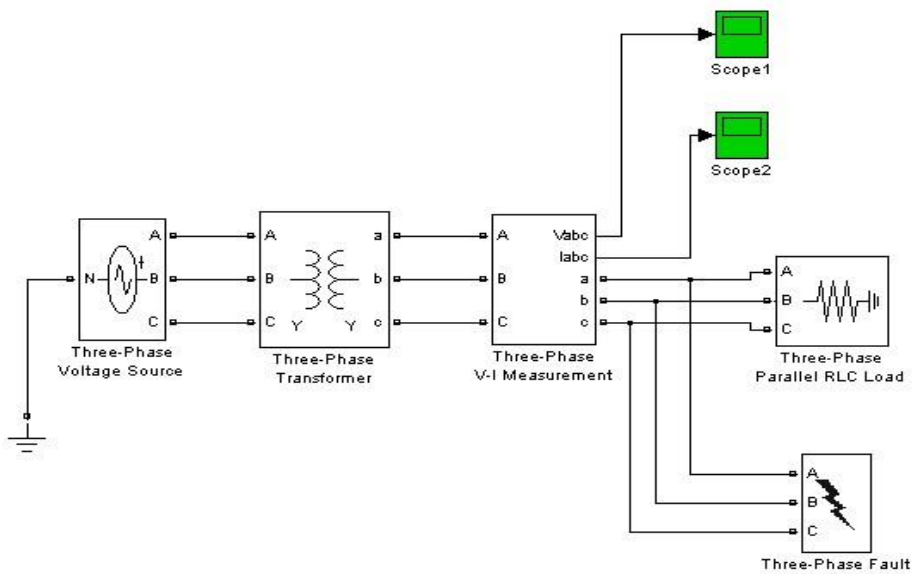


Fig: 1.11 - Three phase fault Analysis

Faulted Phase : Double Line to Ground  
 Transition state : 1 0 1  
 Transition Time : 0 0.05 0.1 0.2

Nominal  $\phi$  to  $\phi$  voltage : 220 V  
 Active Power : 100 W  
 Nominal Frequency : 50 Hz

The faulted segment for a LLG fault is shown in Fig. 1.11 where it is assumed that the fault has occurred at node  $k$  of the network. In this the line to line got shorted through the impedance  $Z_f$  to the ground. Since the system is unloaded before the occurrence of the fault for the phase-a current. Therefore

$$I_{fa0} = \frac{1}{3}(I_{fa} + I_{fb} + I_{fc}) = \frac{1}{3}(I_{fb} + I_{fc}) \quad 4.1$$

$$\Rightarrow 3I_{fa0} = I_{fb} + I_{fc}$$

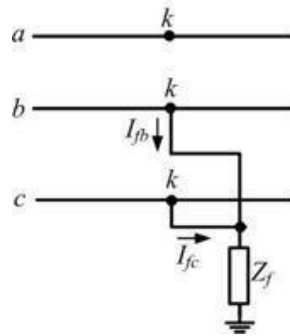


Fig. 1.12 Representation of LLG fault.

Also voltages of phases b and c are given by

Therefore 
$$V_{kb} = V_{kc} = Z_f(I_b + I_c) = 3Z_f I_{fa0} \quad 4.2$$

$$V_{k2012} = C \begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} V_{ka} + 2V_{kb} \\ V_{ka} + (\alpha + \alpha^2)V_{kb} \\ V_{ka} + (\alpha + \alpha^2)V_{kb} \end{bmatrix} \quad 4.3$$

We thus get the following two equations from (4.3)

$$V_{k21} = V_{k22} \quad 4.4$$

$$4.5 \quad 3V_{k20} = V_{ka} + 2V_{kb} = V_{k20} + V_{k21} + V_{k22} + 2V_{kb}$$

Substituting (4.2) and (4.4) in (4.5) and rearranging we get

$$4.6 \quad V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0}$$

Also since  $I_{fa} = 0$  we have

$$4.7 \quad I_{fa0} + I_{fa1} + I_{fa2} = 0$$

The Thevenin equivalent circuit for LLG fault is shown in Fig. 1.13 From this figure we get

The zero and negative sequence currents can be obtained using the current divider principle as

$$I_{fa1} = \frac{V_f}{Z_{kk1} + Z_{kk2} \parallel (Z_{kk0} + 3Z_f)} = \frac{V_f}{Z_{kk1} + \frac{Z_{kk2}(Z_{kk0} + 3Z_f)}{Z_{kk2} + Z_{kk0} + 3Z_f}}$$

current

$$4.10 \quad \begin{aligned} I_{fa0} &= -I_{fa1} \left( \frac{Z_{kk2}}{Z_{kk2} + Z_{kk0} + 3Z_f} \right) \\ I_{fa2} &= -I_{fa1} \left( \frac{Z_{kk2}}{Z_{kk2} + Z_{kk0} + 3Z_f} \right) \end{aligned}$$

4.9

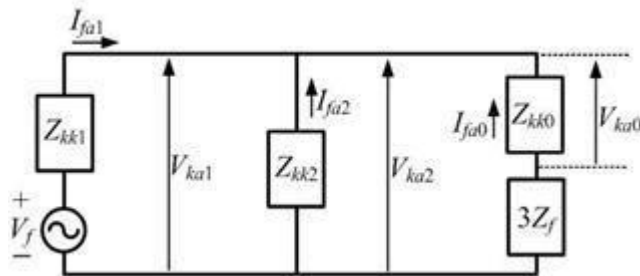


Fig. 1.13 Thevenin equivalent of a LLG fault

**Result:** We have successfully studied the three phase fault on Double Line to Ground fault and obtain the voltage and current waveforms shown below;

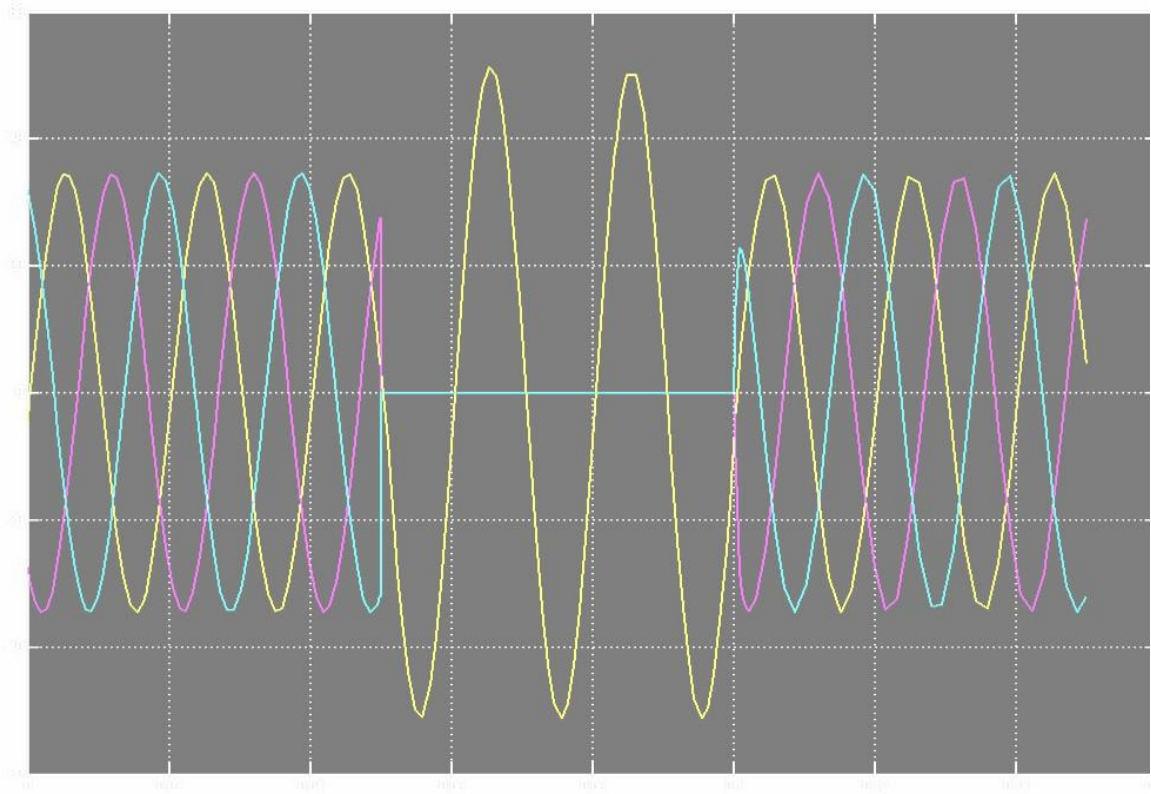


Fig: 1.14- Voltage Waveform of Double Line to Ground Fault

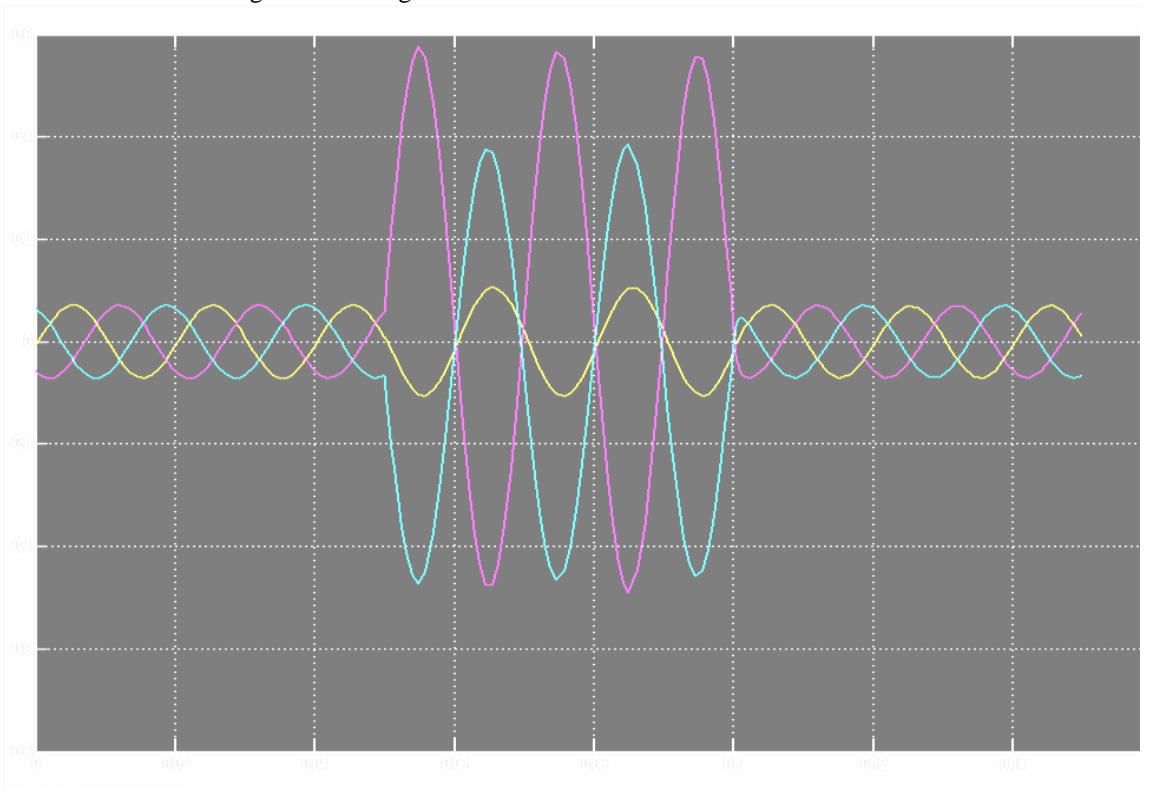


Fig: 1.15 – Current Waveform of Double Line to Ground Fault

## D. THREE PHASE FAULT

This type of faults occurs very rarely. These types of faults occur when all the three phases get in touch with each other. These types of faults occur in power system due to various reasons like lightning strikes, falling of branches or trees on the transmission lines, , insulation failures and other electrical or mechanical causes. All these are collectively called faults in power systems.

A fault usually results in high current flowing through the lines and if adequate protection is not taken, may result in damages in the power apparatus.

Faulted Phase : L-L-L fault on phase A-B-C  
 Transition state : 1 0 1  
 Transition Time : 0 0.05 0.1 0.2  
 Nominal  $\phi$  to  $\phi$  voltage : 220 V  
 Active Power : 100 W  
 Nominal Frequency : 50 Hz

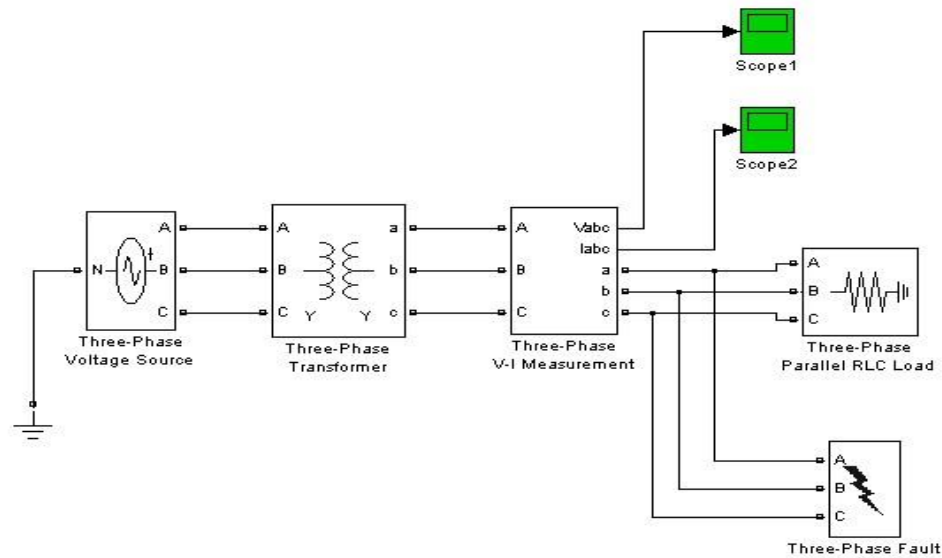


Fig: 1.16-Three phase fault Analysis

**Result:** We have successfully studied the three phase fault ((L-L-L fault on phase A-B-C) and obtain the voltage and current waveforms. The current waveform of three phase fault and the voltage waveform of three phase fault are shown below;



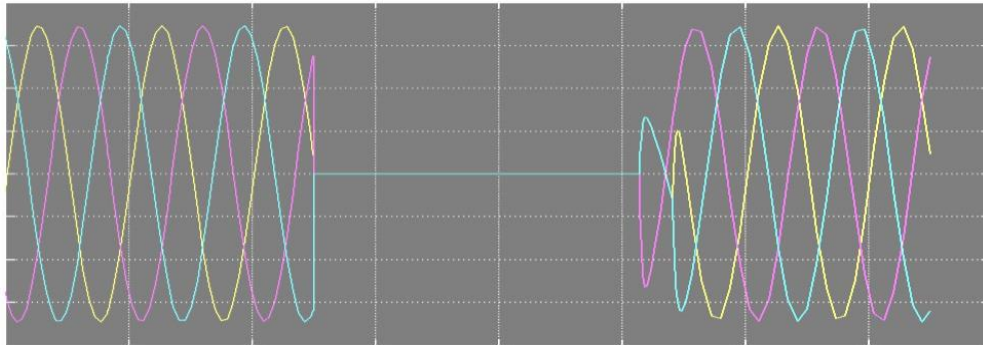


Fig: 1.17 –Voltage Waveform of three phase fault

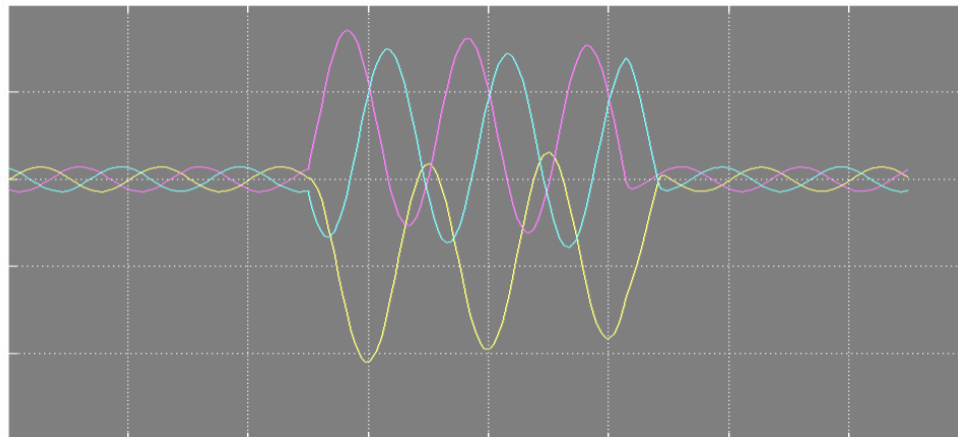


Fig: 1.18–Current Waveform of three phase fault

## **EXPERIMENT # 2**

### **OBJECT:**

Load Flow Analysis for a 3 to 6 bus system, in MATLAB, using the following methods:

- (i) Gauss Seidal Method
- (ii) Newton Raphson
- (iii) Fast Decoupled Method

### **APPARATUS REQUIRED:**

#### **A. Newton Raphson**

Suppose we want to find the value of  $x$  that maximizes some twice continuously differentiable

function  $f(x)$  Recall

$$f(x+h) \approx a+bh+\frac{1}{2}ch^2$$

where  $a=f(x)$ ,  $b=f'(x)$  and  $c=f''(x)$ . This implies

$$f'(x+h) \approx b+ch$$

The first order condition for the value of  $h$  (denoted  $h$ ) that maximizes  $f(x+h) = 0 =$

$$b+ch$$

$$b$$

Which implies  $h = -\frac{b}{c}$ . In other words, the value that maximizes the second order Taylor approximation to  $f$  at  $x$  is

$$\begin{aligned} x+h &= -\frac{b}{c} \\ &= -x \frac{1}{f''(x)} f'(x) \end{aligned}$$

With this in mind we can specify the Newton Raphson algorithm for 1 dimensional function optimization.

### The Newton Raphson Algorithm in $k$ Dimensions

Suppose we want to find the  $x \in \mathbb{R}^k$  that maximizes the twice continually differentiable function  $f:$

$$\mathbb{R}^k \rightarrow \mathbb{R}.$$

Recall

$$f(x+h) \approx a + b'h + \frac{1}{2}h'Ch'$$

Where  $a = f(x)$ ,  $b = \nabla f(x)$ , and  $C = D^2 f(x)$ . Note that  $C$  will be symmetric. This implies

$$\nabla + \nabla f(x) \quad b \quad Ch$$

Once again, the first order condition for a maximum is

$$0 = b + Ch$$

Which implies that

$$h = -C^{-1}b$$

In other words, the vector that maximizes the second order Taylor approximation to  $f$  at  $x$  is

$$\begin{aligned} x+h &= -\frac{1}{C} \nabla f(x) \\ &= -x(D^2 f(x))^{-1} \nabla f(x) \end{aligned}$$

With this in mind we can specify the Newton Raphson algorithm for  $k$  – dimensional function optimization.

## B. Fast Decoupled Method

As the FDPFM is derived from the Newton – Raphson we will start from the matrix representation for NR, apply some simplification and approximation, to reach the equation of the FDPFM. The matrix representation of the NR method is;

$$\begin{bmatrix} P \\ Q \end{bmatrix} = -JL^{-1}V \quad (1)$$

Where

$$H_{ii} = \sum_{j \neq i} |V_i| |V_j| Y_{ij} \sin(\delta_{ij} - \delta_i - \delta_j) \quad (2)$$

$$H_{ij} = -|V_i| |V_j| Y_{ij} \sin(\delta_{ij} - \delta_i - \delta_j) \quad (3)$$

And

$$N_{ii} = 2 \sum_{j \neq i} |V_i| |V_j| Y_{ij} \cos(\delta_{ij} - \delta_i - \delta_j) \quad (4)$$

$$N_{ij} = |V_i| |V_j| Y_{ij} \cos(\delta_{ij} - \delta_i - \delta_j) \quad (5)$$

$$J_{ii} = \sum_{j \neq i} |V_i| |V_j| Y_{ij} \cos(\delta_{ij} - \delta_i - \delta_j) \quad (6)$$

$$J_{ij} = -|V_i| |V_j| Y_{ij} \cos(\delta_{ij} - \delta_i - \delta_j) \quad (7)$$

$$L_{ii} = 2 \sum_{j \neq i} |V_i| |V_j| Y_{ij} \sin(\delta_{ij} - \delta_i - \delta_j) \quad (8)$$

$$L_{ij} = |V_i| |Y_{ij}| \sin(\theta_{ij} + \delta_i - \delta_j) \quad (9)$$

Now, for typical power system branches:

$$X/R \gg 1 \quad \text{and} \quad \theta_{ij} < 20^\circ \quad (10)$$

These two approximations will cause a weak coupling between  $\Delta P$  and  $\Delta V$ , and between  $\Delta Q$  and  $\Delta \delta$ , hence N and J entries of the initial matrix of (1) can be ignored leading to the following decoupled equations:

$$[\Delta P] = [H][\Delta \delta] \quad (11)$$

$$[\Delta Q] = [L][\Delta V] \quad (12)$$

Now, the diagonal elements of H according to Stott and Alsac [4] can be written as:

$$H_{ii} = -Q_i - |V_i|^2 B_{ii} \quad (13)$$

Where  $B_{ii} = |Y_{ii}| \sin \theta_{ii}$  is the imaginary part of the diagonal elements of the bus admittance matrix  $Y_{bus}$ .

Further simplifications can be applied to equation (12), by considering  $B_{ii} \gg Q_i$  and  $|V_i|^2 \approx |V_i|$  yielding to the following simplified  $H_{ii}$ :

$$H_{ii} = -|V_i| B_{ii} \quad (14)$$

Also, as under normal operating conditions  $\delta_j - \delta_i$  is quite small, thus  $\theta_{ij} - \delta_i + \delta_j \approx \theta_{ii}$ , and  $|V_j| \approx 1$ , the off-diagonal elements of the matrix H can be written as:

$$H_{ij} = -|V_i| B_{ij} \quad (15)$$

Similarly, the diagonal elements of the L matrix can be written as:

$$L_{ii} = -|V_i| B_{ii} \quad (16)$$

And its off-diagonal elements as:

$$L_{ij} = -|V_i| B_{ij} \quad (17)$$

Applying these assumptions to equations (11) and (12) we get:

$$\frac{\Delta P}{|V_i|} = -B' \Delta \delta \quad (18)$$

$$\frac{\Delta Q}{|V_i|} = -B'' \Delta |V| \quad (19)$$

where  $B'$  and  $B''$  are the imaginary part of the bus admittance matrix  $Y_{bus}$ , such that  $B'$  contains all buses admittances except those related to the slack bus, and  $B''$  is  $B'$  deprived from all voltage-controlled buses related admittances.

Finally, all these approximations and simplifications lead to the following successive voltage magnitude and voltage angle updating equations:

where  $B'$  and  $B''$  are the imaginary part of the bus admittance matrix  $Y_{bus}$ , such that  $B'$  contains all buses admittances except those related to the slack bus, and  $B''$  is  $B'$  deprived from all voltagecontrolled buses related admittances. Finally, all these approximations and simplifications lead to the following successive voltage magnitude and voltage angle updating equations:

**CONCLUSION:** We successfully have done the algorithm of Gauss Seidal Method, Newton Raphson &Fast Decoupled Method.

# EXPERIMENT # 3

## **OBJECT:**

Study of voltage security in power system

## **THEORY:**

Voltage stability is the ability of a power system to maintain acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance. A system enters a state of voltage instability when a disturbance, increase in load demand, or change in system condition cause a progressive and uncontrollable decline in voltage. The main factor causing voltage instability is the inability of the power system to meet the demand for reactive power. Voltage collapse is the process or sequence of events accompanying voltage instability which leads to a low unacceptable voltage profile in a significant part of the system.

For maintaining voltage security within the system, the following need to be monitored from time to time

1. Available voltage security margin
2. The most dangerous stresses in the system leading to voltage collapse
3. Worst-case contingencies resulting in voltage collapse and/or contingencies with insufficient voltage stability margin
4. Contingency ranking according to a severity index for voltage stability related system problems
5. Weakest elements within the grid and the regions most affected by potential voltage problems
6. Controls to increase the available stability margin and avoid instability
7. Information about voltage problems at the look-ahead operating conditions and for the worstcase contingencies (contingencies with large severity ranks) that may appear in the future
8. A real-time dispatcher's situational awareness-type wide area graphic and geographic displays.

## **MODES OF OPERATION**

1. Real-Time Modes - Under the 'Real Time Operations Mode', a real time assessment of the most current state estimation is done.

On the other hand, in the 'Real Time Look-Ahead Mode' we perform a 2-hour "look-ahead" predictive assessment by applying planned outage information available and load forecast over the next 2 hours.

2. Study Mode - Study mode offers off-line analysis capabilities on either the real-time data or on modified version of real-time solved cases.

**Such study cases are:**

Real-Time Voltage Security Assessment (RTVSA) solved cases archived overtime within the Flat Files Storage (under Central Server)

- (i) Modified versions of the above mentioned real-time solved cases to study hypothetical scenarios. For instance, a study mode user may extract a previously archived RTVSA solved case from the Flat Files Storage, remove one or more transmission lines, manually specify stressing directions, resolve using the RTVSA simulation capabilities and perform a complete voltage security assessment, and export this as a new “study case” to the central server if so desired.

**REAL-TIME VOLTAGE SECURITY ASSESSMENT (RTVSA) CAPABILITIES**

The RTVSA application shall offer the following categories of functional capabilities:

- 1) Contingency screening and ranking with respect to voltage limit violations or loading margins associated with known stressing direction.
- 2) Wide area monitoring capabilities offering real time situational awareness to the operators on key indicators that are closely associated with voltage security
- 3) Real time voltage stability analysis with known stressing direction
- 4) Quantify the efficacy of reactive power support at the most effective buses in terms of their sensitivities
- 5) Rank available corrective controls based on their
- 6) Identify the weak elements within the system associated with the one-dimensional stressing
  - a) Develop and update voltage security regions offline on demand based on a set of predefined stressing
  - b) Real time voltage security assessment with respect to the multidirectional stressing
  - c) Suggest appropriate controls to enhance margin to the boundary

**DATA DESCRIPTION**

The following are details on the required list of data:

- **Detailed Network Model** : Contains information in a volume sufficient for detailed power flow simulations, under the CA ISO standards, i.e., branch information (connectivity data, line impedance), breaker status, etc.
- **System Component Status Information** : Includes current status of generators, transmission circuits, transformers, switching devices, and other components.

- **Available Power System Controls and their priorities** : These include information of -
  - Tap Changers
  - Static VAR Compensator (SVC)
  - Fixed and Controllable Shunt - Generator Redispatch, etc.
- **Limits (Voltage, Thermal, MVar, Others)** : Consists of operational limits of system facilities/components that are to be specified in appropriate units, e.g. transformer limits in MVA, line limits in Amps, etc
- **Generator Model** : Required information for generator modeling, such as:
  - MVA ratings
  - $Q_{\max}$ ,  $Q_{\min}$  values
  - Leading and lagging power factor
- **Distributed Slack Bus Information** : Required for governor power flow simulations
- **Low Voltage Load Models** : These models (static characteristics) should cover the low voltage load behavior and voltage collapse situations.
- **HVDC Models & Control Schemes** ○
- Contingency List** : Consists of -
  - All (N-1) and some (N-2) contingencies, or
  - User specified contingency list
  - Any Remedial Action Schemes (RASs) associated with these contingencies ○ **Stressing**
- Directions & Descriptor Variables** : Contains -
  - Generator dispatch sequence & pattern
  - Load stress pattern

## **SPECIAL PROTECTION SCHEMES/REMEDIAL ACTION SCHEMES**

During the system stressing process and contingency analysis, it is required for the RTVSA tool to automatically trigger Remedial Action Schemes (RAS) or Special Protection Schemes (SPS) to provide realistic voltage stability margins.

## **VOLTAGE SECURITY ASSESSMENT**

The display capabilities under this category demonstrate results of the Voltage Security Assessment tool under the look-ahead scenario with respect to key stressing direction(s). Such scenarios may be based on current operating conditions or under the worst case contingency. These illustrate voltage security conditions and metrics that help users study voltage stability and take decisions to prevent adverse situations. These capabilities include, but are not limited to: - Real and reactive loading margins

- ✦ Margin at base case to point of collapse (POC)



- ✦ Margin under worst case contingency base case to POC
- Contingency ranking based on severity index (voltage margin, loading margin, etc.) -  
Operating nomograms i.e. the chart representing numerical relationships.
- Distance to instability
- Weak elements information
- Corrective actions (preventive control, enhancement control)

**CONCLUSION:** We successfully study the voltage security in power system.

## **EXPERIMENT # 4**

### **OBJECT:**

Study of overload security analysis and obtain results for the given problem using MATLAB or any software.

### **THEORY:**

The recent evolution of the electric power industry has brought about new needs in terms of assessing the reliability of the transmission system. Perhaps the most important of these include its accurate assessment and the need to integrate reliability into economic decision making. These needs exist at the operational level. In this paper, we address them in terms of the one year planning problem. There are today a number of commercial software packages that include the influence of circuit overload in a reliability assessment scheme. All programs develop probabilistic indices characterizing the power system reliability level, although some use analytical approaches, sometimes called contingency enumeration, while others use Monte Carlo simulation. Some of the most well known in North America include TRELSS, TPLAN, PROCOSE, and CREAM. The approaches for assessing circuit overload for planning purposes used in these and other programs have rested on two main assumptions. These are:

1. The circuit overload reliability level is indicated by a measurement of the amount of load shed necessary to avoid circuit overload; loss of load probability (LOLP) and expected unsaved energy (EUE) are two of the most common measurements used.
2. Measurements taken on one or a limited number of selected base cases are sufficient to indicate the reliability of the system.

### **SEQUENTIAL MEAN VARIANCE (SMV) MODEL DESCRIPTION**

The sequential mean variance (SMV) model first uses the expected annual load curve, sampled hourly,<sup>1</sup> to arrange the maintenance and unit commitment schedules, then employs time invariant

variances to represent normally distributed load uncertainties. The expected annual load curve can be obtained from load forecasting, or it can be obtained from the load curve of the previous year, with an appropriate scaling to account for load growth. There are various methods to identify the maintenance, unit commitment schedule and load forecasting error. We propose a feasible one for each in order to show the effectiveness of our overall framework.

For the maintenance schedule arrangement, we apply the equal LOLP criterion by utilizing the effective load carrying capacity. For the unit commitment, we employ the priority list method based on the piecewise linear fuel consumption curve, considering hydro-thermal coordination. Our method for unit commitment also results in an economic dispatch calculation for the generators in each hour. This provides us an hourly base case for which we can solve the power flow equations.

For load forecasting error identification, we first employ time series analysis to identify the structure and parameters of an ARIMA (autoregressive integrated moving average) model used to represent the load series. This provides a load value for each hour. We assume that each hourly load value used in our trajectory has associated with it some error. This error characterizes the potential for deviation away from the load forecast for which the system coordinator (perhaps the independent system operator) is unable to make effective and economically efficient adjustments. We assume that such adjustments would be possible given more than a one-day advance warning by using the day-ahead electricity market, but they would not be possible for advance warning less than one day. Therefore, we use the estimated error of a one-day forecast in our work. This estimated error is computed by averaging the errors of a day-ahead load forecast as compared with historical data, over one year. Other reasoning could be used to identify this error, if appropriate.

For each hour, we use the load curve value as the mean load and the error as the standard deviation of the load in order to represent normally distributed load uncertainty. Allocation among buses is done according to assumed load sharing factors, but statistical correlation between loads may also be employed if data is available.

The result of this modeling effort is a series of 8760 samples, one per hour over a year, for which we know the committed generation units, their dispatch and a probability density function (pdf) for the load. We denote the series of 8760 samples, one per hour over a year, for which we know the committed generation units, their dispatch and a probability density function (pdf) for the load. We denote the series of 8760 samples as  $\Omega$ . Let's consider a single hour,  $h$ , a single contingency state,  $s$ , and a single branch,  $b$ , denoted by  $\Pr(I_b|h, s, \Omega)$  and if we have a function  $Risk(TOL|I_b)$  which gives the expected monetary impact of each flow  $I_b$  on branch  $b$ , the component risk, then we can compute the thermal overload (TOL) risk for the particular contingency state,  $s$ , in hour,  $h$ , as

$$Risk(TOL|h, s, b, \Omega) = \int_{-\infty}^{\infty} \Pr(I_b|h, s, \Omega) Risk(TOL|I_b) dI_b.$$

The total risk for this branch in hour over all contingency states is then

$$Risk(TOL|h, b, \Omega) = \sum_s Pr(s) Risk(TOL|h, s, b, \Omega)$$

From (2), we may sum over all branches to obtain total risk for a particular hour, or we may sum over all hours to obtain the cumulative risk for a particular branch. These kinds of calculations reflect the decomposition capability of this approach and are attractive for identifying the reasons for high risk. In addition, we may evaluate total cumulative risk as

$$Risk(TOL|\Omega) = \sum_h \sum_b Risk(TOL|h, b, \Omega).$$

These calculations, together with those required to obtain, are referred to as thermal overload risk assessment. Its use, together with the trajectory development, is illustrated in Fig. 1. From this figure, we also observe that the results of assessing the risk of a trajectory are used to update the unit commitment (or other operating policies) and/or the facility plan as needed to reduce risk. These updates correspond to different decisions and are therefore re-evaluated to determine their effect on risk.

## COMPONENT RISK

Equation (1) requires  $Risk(TOL|I_b)$ , which is the expected monetary impact on branch due to overload given the flow on branch,  $b$ . If branch  $b$  is a transmission line, then, depending on the weather conditions, conductor type, and flow duration, the flow  $I_b$  causes conductor heating which can result in one or both of the following:

- Loss of clearance due to sag: Here, the thermal expansion of the conductor results in sag. In the worst case, the line can touch an underlying object, resulting in a permanent fault and subsequent outage.
- Loss of strength due to annealing: Annealing, the recrystallization of metal, is a gradual and irreversible process when the grain matrix established by cold working is consumed causing loss of tensile strength. In [12], we have shown how to use weather statistics to obtain  $f(\theta | I_b)$ , the pdf for conductor temperature,  $\theta$ . This can be used to obtain the desired risk expression as

$$Risk(TOL|I_b) = \int_{\theta} f(\theta|I_b)[Im_{L1}(\theta) + Im_{L2}(\theta)] d\theta$$

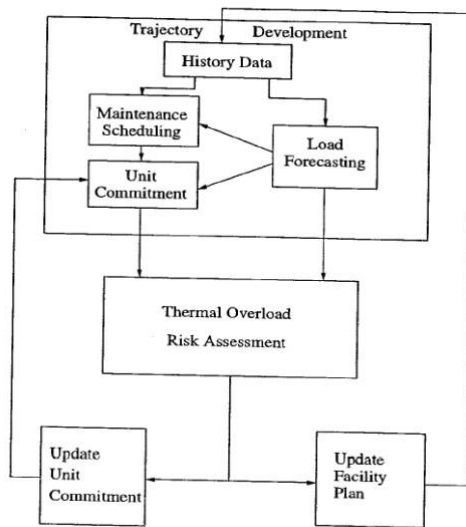


Fig. 1. Annual thermal overload risk assessment framework.

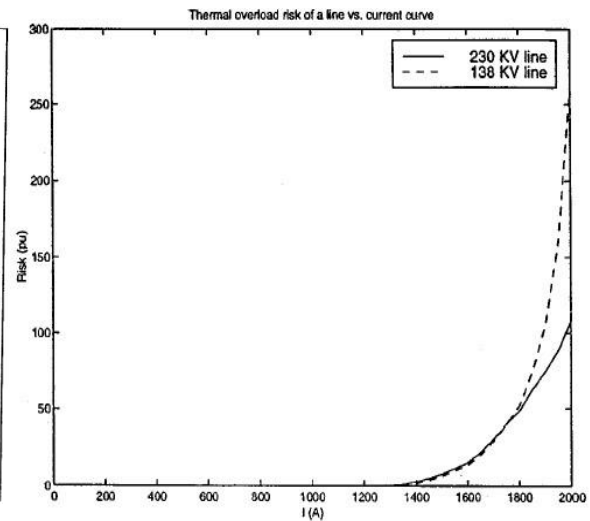


Fig. 2. The thermal overload risk vs. line current curve.

Where  $ImL_1(\theta)$  and  $ImL_2(\theta)$  and express the monetary impact on the transmission line of sag and annealing, respectively, as a function of conductor temperature, also described in . Equation (4) can be evaluated for a range of flows, resulting in a component risk curve for branch b, as shown in Fig. 2, where the pdfs for ambient temperature and wind speed are typically chosen. The same pdf for ambient temperature is also used in transformer risk assessment. The two curves for Fig. 2 are per unitized on a base equal to the cost of reconductoring 1 mile of the line. This base value is estimated based on discussions with utility engineers as \$108 000 for a 230 kV line and \$60 606 for a 138 kV line.

If branch b is a transformer, then depending on the ambient temperature, transformer type and aging rates of insulation materials (paper and oil), and flow duration, higher flow  $I_b$  causes winding hottest spot temperature to increase which can result in transformer loss of life and/or failure. We can then use an expression just like (4) to evaluate the thermal overload risk, except here,  $\theta$  represents the hottest spot temperature, and  $ImL_1$  and  $ImL_2$  represent the monetary impact on the transformer of failure and loss of life, respectively, as described in and . With these modifications, we can evaluate eqt. (4) for a range of flows, resulting in a component curve for branch b, as shown in Fig. 3. Here, 1.0 pu risk equals the cost to rebuild the transformer. It is chosen to be \$1 000 000 in and . The risk evaluation of both lines and transformers, as a function of loading, must also account for the impact on the system caused by outage of the circuit due to high loading. In some cases, circuit outage has very little system impact, but in other cases, it results in cascading leading to islanding and/or widespread outages. There are various approaches that one can take to evaluate this impact. For example, one could detect the extent of cascading overloads by performing a series of power flow solutions, each time removing any additional overloaded circuits. More rigorous analysis would require representation of system dynamics. Here, we have accounted for this impact very simply, but conservatively, by assigning any

circuit failure, as represented by  $I_{mL1}$  in (4), to have an impact  $K$  times the cost of replacing the equipment, where  $K$  is a very large number. In the component risk curves shown in Figs. 2 and 3, we have assigned  $K=100$ . The practical result of this is that circuit loadings causing any significant probability of failure contribute very large risk. The component risk curves for both lines and transformers clearly depend on the weather statistics. One can significantly enhance the analysis accuracy by using different weather descriptions for different times of the day and for different seasons.

For example, one might divide the 24 hour period into four 6 hour intervals, late morning, afternoon, evening, early morning, and one might divide the year into the four seasons of winter spring, summer and fall. Of course, this would require 16 component risk curves for every line and transformer. However, these curves may be computed and stored in advance of the trajectory simulation, so that their number does not affect the processing time. In our work, in order to illustrate the idea with the simplest approach, we have used the same weather statistics for all component risk curves.

## CONCLUSION

We have proposed the sequential mean variance (SMV) model together with a risk index to assess power system reliability over a mid-term planning period. We have shown that the SMV model enables assessment of loading periods and inter-temporal affects that may not be captured by so-called snapshot models. Yet, it does so with reduced computational requirements relative to the sequential Monte Carlo model.

The strength of the method lies in its ability to identify a-priori high-risk situations encountered during an expected trajectory of yearly operating conditions, and then to avoid or mitigate these conditions using short-term operational or reinforcement measures (see footnote 1). This is in contrast to longterm facility planning needs, where one thinks of performing design that is robust to a wide range of possible trajectories. The risk index used in the hourly assessment provides a compact evaluation of the hour's reliability level for overload that does not require the representation of the operator's load shedding policy, considered here to be a decision which could be assessed by the risk index. This risk assessment is performed based on linearization around the operating point and convolution between random variables. The risk index can be presented as cumulative over time, it can be decomposed according to which agent incurs it, and it can be assigned to the agent that causes it.

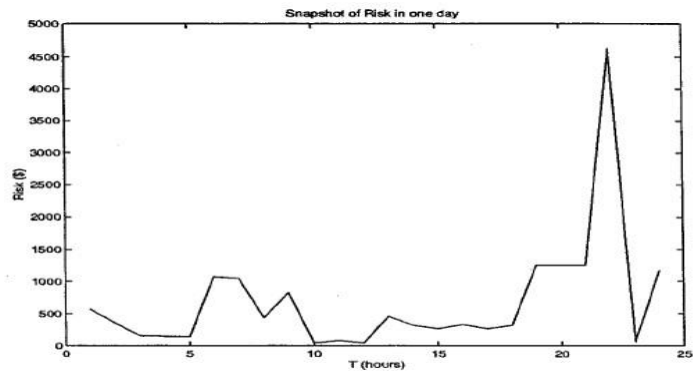


Fig. 8. Risk curve of the day with the peak risk hour.

**RESULT:** We successfully study overload security analysis in power system.

# EXPERIMENT # 5

## OBJECT:

Study the various methods of Economic Load Dispatch

## **THEORY:**

Economic load dispatch (ELD) is an important function in power system planning and operation. ELD solutions are found by solving the conventional load flow equations while at the same time minimizing fuel costs. The resulting optimization problem has nonlinear constraints from the load flow nodal equations and simple bound constraints on the variables from the load bus voltage magnitudes.

Methods of Economic Load Dispatch:

- 1) **Lambda Search** : In 1962, Carpentier introduced a generalized nonlinear programming formulation of the economic dispatch problem, including voltage and other operating constraints. This formulation was later named the Optimal Power Flow (OPF) problem.
- 2) **Gradient Algorithm** : In 1968, Dommel and Tinney introduced a reduced gradient steepest descent algorithm to solve the optimization problem. This algorithm has two drawbacks: slow convergence with the steepest descent direction, and ill conditioning resulting from the penalty functions associated with the inequality constraints.
- 3) **Newton's Method**

For solving the problem with economic dispatch we first need to define or formulate the problem.

### **Economic Load Dispatch (ELD) problem formulation**

The ELD problem is considered as a general minimization problem with constraints, and can be written in the following form:

$$\begin{aligned} \text{Minimize} \quad & f(x) & (1) \quad \text{Subject} \\ \text{to:} \quad & g(x) = 0 & (2) \\ & h(x) \leq 0 & (3) \end{aligned}$$

$f(x)$  is the objective function,  $g(x)$  and  $h(x)$  are respectively the set of equality and inequality constraints.  $x$  is the vector of control and state variables. The control variables are generator active and reactive power outputs, bus voltages, shunt capacitors/reactors and transformers tap-setting. The state variables are voltage and angle of load buses.

## Objective function

The objective function for the ELD reflects the costs associated with generating power in the system. The quadratic cost model is used. The objective function for the entire power system can then be written as the sum of the quadratic cost model for each generator:

$$f(x) = \sum_{i=1}^{ng} a_i + b_i P_{gi} + c_i P_{gi}^2 \quad [\$/h] \quad (4)$$

Where,  $ng$  is the number of thermal units,  $P_{gi}$  is the active power generation at unit  $i$  and  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients of the  $i^{\text{th}}$  generator.

## Equality constraints

The equality constraints  $g(x)$  of the ELD problem are represented by the power balance constraint, where the total power generation must cover the total power demand and the power loss. This implies solving the load flow problem, which has equality constraints on active and reactive power at each bus as follows [4]:

$$\begin{aligned} P_i &= P_{gi} - P_{di} = \sum_{j=1}^n V_i \cdot V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ Q_i &= Q_{gi} - Q_{di} = \sum_{j=1}^n V_i \cdot V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \end{aligned} \quad (5)$$

where:  $i=1,2,\dots, n$  and  $\theta_{ij} = \theta_i - \theta_j$

$P_i, Q_i$ : injected active and reactive power at bus  $i$

$P_{di}, Q_{di}$ : active and reactive power demand at bus  $i$

$V_i, \theta_i$ : bus voltage magnitude and angle at bus  $i$

$G_{ij}, B_{ij}$ : conductance and susceptance of the  $(i,j)$  element in the admittance matrix.

## Inequality constraints

The inequality constraints  $h(x)$  reflect the limits on physical devices in the power system as well as the limits created to ensure system security:

- Upper and lower bounds on the active and reactive generations:

$$P_{gi \min} \leq P_{gi} \leq P_{gi \max}, \quad Q_{gi \min} \leq Q_{gi} \leq Q_{gi \max} \quad (6)$$

- Upper and lower bounds on the tap ratio ( $t$ ) and phase shifting ( $\alpha$ ) of variable transformers:

$$t_{ij \min} \leq t_{ij} \leq t_{ij \max}, \quad \alpha_{ij \min} \leq \alpha_{ij} \leq \alpha_{ij \max} \quad (7)$$

- Upper limit on the active power flow ( $P_{ij}$ ) of line  $i-j$ :

$$|P_{ij}| \leq P_{ij \max} \quad (8)$$

$$P_{ij} = \left| -G_{ij} V_i^2 + G_{ij} V_i V_j \cos(\theta_i - \theta_j) + B_{ij} V_i V_j \sin(\theta_i - \theta_j) \right| \quad (9)$$

- Upper and lower bounds on the bus voltage magnitude:

$$V_{i \min} \leq V_i \leq V_{i \max} \quad (10)$$



The ELD problem formulated above can be solved by any of the above listed three methods. This is explained as follows:

### 1) LAMBDA SEARCH METHOD:

This method can be described for 2 cases – neglecting or including losses.

**ELD Neglecting Losses:** Let us consider a system of  $N$  thermal-generating units connected to a single bus-bar serving a received electrical load  $P_{load}$ . The input to each unit, shown as  $F_i$ , represents the cost rate of the unit. The output of each unit,  $P_i$ , is the electrical power generated by that particular unit. The total cost rate of this system is, of course, the sum of the costs of each of the individual units. The essential constraint on the operation of this system is that the sum of the output powers must equal the load demand. That is, an objective function,  $F_T$ , is equal to the total cost for supplying the indicated load. The problem is to minimize  $F_T$  subject to the constraints specified above note that any transmission losses are neglected and any operating limits are not explicitly stated when formulating this problem.

$$F_T = F_1 + F_2 + F_3 + \dots + F_N = \sum_{i=1}^N F_i(P_i)$$

$$\phi = 0 = P_{load} - \sum_{i=1}^N P_i$$

This is solved by Lagrangian multiplier method. Let  $\lambda$  be the Lagrangian multiplier, then the function becomes,

$$\mathcal{L} = F_T + \lambda\phi$$

Finding the minimum value of the above function gives the following equation,

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{dF_i(P_i)}{dP_i} - \lambda = 0$$

After solving the above equation it gives following condition for economic dispatch neglecting losses

$$\frac{dF_i}{dP_i} = \lambda \quad N \text{ equations}$$

$$P_{i,\min} \leq P_i \leq P_{i,\max} \quad 2N \text{ inequalities}$$

$$\sum_{i=1}^N P_i = P_{load} \quad 1 \text{ constraint}$$

When we recognize the inequality constraints, then the necessary conditions may be expanded slightly as shown in the following equations :

$$\frac{dF_i}{dP_i} = \lambda \quad \text{for } P_{i,\min} < P_i < P_{i,\max}$$

$$\frac{dF_i}{dP_i} \leq \lambda \quad \text{for } P_i = P_{i,\max}$$

$$\frac{dF_i}{dP_i} \geq \lambda \quad \text{for } P_i = P_{i,\min}$$

**ELD Considering Losses:** In this case the power generated will be equal to the sum of power demand and power losses,  $P_{\text{loss}}$ . Thus the following equation follows:

$$P_{\text{load}} + P_{\text{loss}} - \sum_{i=1}^N P_i = \phi = 0 \quad (11)$$

Now by Lagrange's multiplier method we get the following equation

$$\mathcal{L} = F_T + \lambda \phi$$

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{dF_i}{dP_i} - \lambda \left( 1 - \frac{\partial P_{\text{loss}}}{\partial P_i} \right) = 0 \quad (12)$$

$$\frac{dF_i}{dP_i} + \lambda \frac{\partial P_{\text{loss}}}{\partial P_i} = \lambda \quad (13)$$

Equations (11) to (13) are known as coordination equations.

The procedure for solution by this method is defined below:

Step 1: Pick a set of starting values for  $P_1$ ,  $P_2$ , and  $P_3$  that sum to the load.

Step 2: Calculate the incremental losses  $dPL/dP_i$ , as well as the total losses.

Step 3: Assume the value of  $\lambda$  greater than the largest intercept of the cost functions. Evaluate values of  $P_1, P_2, \dots, P_n$  by (11), (12) and (13).

Step 4: Check whether generations  $P_1, P_2, \dots, P_n$  are within the prescribed limits.

If  $P_i > P_{\text{max}}$ , then set  $P_i = P_{\text{max}}$ , and distribute the remaining load between the remaining units by equation (11), or

If  $P_i < P_{\text{min}}$ , then set  $P_i = P_{\text{min}}$ , and distribute the remaining load between the remaining units by equation (11)

Step 5: Check if value of equation (11)  $< \epsilon$  (defined tolerance), then increase value of  $\lambda$  slightly by  $\lambda + \Delta\lambda$ , otherwise decrease it by  $\lambda - \Delta\lambda$ , and go to step 2.

If the solution of (11) is within specified tolerance limit, then go to step 6.

Step 6: Print the results of  $P_i$  and  $P_{\text{loss}}$ , and stop.

## 2) GRADIENT ALGORITHM METHOD :

Note that the lambda search technique always requires that one be able to find the power output of a generator, given an incremental cost for that generator. In the case of a quadratic function for the cost function, or in the case where the incremental cost function is represented by a piecewise

linear function, this is possible. However, it is often the case that the cost function is much more complex, such as the one below:

$$F(P) = A + BP + CP^2 + D \exp\left[\frac{(P - E)}{F}\right]$$

In this case, we shall propose that a more basic method of solution for the optimum be found. This method works on the principle that the minimum of a function,  $f(x)$ , can be found by a series of steps that always take us in a downward direction. From any starting point,  $x^0$ , we may find the direction of “steepest descent” by noting that the gradient of  $f$ , i.e.,

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

Always points in the direction of maximum ascent. Therefore, if we want to move in the direction of maximum descent, we negate the gradient. Then we should go from  $x^0$  to  $x^1$  using:

$$x^1 = x^0 - \nabla f \alpha$$

Where  $\alpha$  is a scalar to allow us to guarantee that the process converges. The best value of  $\alpha$  must be determined by experiment.

### Economic Dispatch by Gradient Search

In the case of power system economic dispatch this becomes:

$$f = \sum_{i=1}^N F_i(P_i)$$

and the object is to drive the function to its minimum. However, we have to be concerned with the constraint function:

$$\Phi = \left( P_{\text{load}} - \sum_{i=1}^N P_i \right)$$

To solve the economic dispatch problem which involves minimizing the objective function and keeping the equality constraint, we must apply the gradient technique directly to the Lagrange function itself. The Lagrange function is:

$$\mathcal{L} = \sum_{i=1}^N F_i(P_i) + \lambda \left( P_{\text{load}} - \sum_{i=1}^N P_i \right)$$

and the gradient of this function is:

$$\nabla \mathcal{L} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial P_1} \\ \frac{\partial \mathcal{L}}{\partial P_2} \\ \frac{\partial \mathcal{L}}{\partial P_3} \\ \frac{\partial \mathcal{L}}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \frac{d}{dP_1} F_1(P_1) - \lambda \\ \frac{d}{dP_2} F_2(P_2) - \lambda \\ \frac{d}{dP_3} F_3(P_3) - \lambda \\ P_{\text{load}} - \sum_{i=1}^N P_i \end{bmatrix}$$

The problem with this formulation is the lack of a guarantee that the new points generated each step will lie on the surface  $\phi$ . We shall see that this can be overcome by a simple variation of the gradient method. The economic dispatch algorithm requires a starting  $\lambda$  value and starting values for  $P_1$ ,  $P_2$ , and  $P_3$ . The gradient for  $\mathcal{L}$  is calculated as above and the new values of  $\lambda$ ,  $P_1$ ,  $P_2$ , and  $P_3$ , etc., are found from:

$$\mathbf{x}^1 = \mathbf{x}^0 - (\nabla \mathcal{L}) \alpha$$

where the vector  $\mathbf{x}$  is:

$$\mathbf{x} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ \lambda \end{bmatrix}$$

### 3) NEWTON'S METHOD

We may wish to go a further step beyond the simple gradient method and try to solve the economic dispatch by observing that the aim is to always drive:

$$\nabla \mathcal{L}_x = 0$$

Since this is a vector function, we can formulate the problem as one of finding the correction that exactly drives the gradient to zero (i.e., to a vector, all of whose elements are zero). We know how to find this, however, since we can use Newton's method. Newton's method for a function of more than one variable is developed as follows.

Suppose we wish to drive the function  $\mathbf{g}(\mathbf{x})$  to zero. The function  $\mathbf{g}$  is a vector and the unknowns,  $\mathbf{x}$ , are also vectors. Then, to use Newton's method, we observe:

$$\mathbf{g}(\mathbf{x} + \Delta \mathbf{x}) = \mathbf{g}(\mathbf{x}) + [\mathbf{g}'(\mathbf{x})] \Delta \mathbf{x} = 0$$

If we let the function be defined as:

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} g_1(x_1, x_2, x_3) \\ g_2(x_1, x_2, x_3) \\ g_3(x_1, x_2, x_3) \end{bmatrix}$$

Then

$$g'(x) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & & \end{bmatrix}$$

which is the familiar Jacobian matrix. The adjustment at each step is then:

$$\Delta \mathbf{x} = -[g'(x)]^{-1} \mathbf{g}(x)$$

Now, if we let the  $g$  function be the gradient vector  $\nabla \mathcal{L}_x$  we get:

$$\Delta \mathbf{x} = -\left[\frac{\partial}{\partial \mathbf{x}} \nabla \mathcal{L}_x\right]^{-1} \Delta \mathcal{L}$$

For our economic dispatch problem this takes the form:

$$\mathcal{L} = \sum_{i=1}^N F_i(P_i) + \lambda \left( P_{\text{load}} - \sum_{i=1}^N P_i \right)$$

and  $\nabla \mathcal{L}$  is as it was defined before. The Jacobian matrix now becomes one made up of second derivatives and is called the Hessian matrix:

$$\left[\frac{\partial}{\partial \mathbf{x}} \nabla \mathcal{L}_x\right] = \begin{bmatrix} \frac{d^2 \mathcal{L}}{dx_1^2} & \frac{d^2 \mathcal{L}}{dx_1 dx_2} & \dots \\ \frac{d^2 \mathcal{L}}{dx_2 dx_1} & \dots & \\ \vdots & \vdots & \\ \frac{d^2 \mathcal{L}}{d\lambda dx_1} & \dots & \end{bmatrix}$$

Generally, Newton's method will solve for the correction that is much closer to the minimum generation cost in one step than would the gradient method.

**CONCLUSION:** We successfully study the various methods of Economic Load Dispatch.